CORRELATION OF LOCAL STRENGTH GRADIENT CRITERIA IN A STRESS CONCENTRATION ZONE WITH LINEAR FRACTURE MECHANICS

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Questions connected with material brittle failure in zones of stress concentration have frequently attracted the attention of researchers. In fact, use in this case of classical strength criteria as a rule gives low values of calculated limiting loads compared with experimental data for specimens with concentrators, particularly if specimens are prepared from brittle structurally inhomogeneous materials of the cast iron type [1, 2], graphite [3, 4], and glass-reinforced plastics [5, 6].

In [1, 5-13] experimental verification and theoretical development of a gradient approach was formulated for the question of material strength with a nonuniform stressed state in the vicinity of concentrators. At the same time in order to evaluate the strength of structural elements with cracks other methods have been developed which then comprised the basis of modern fracture mechanics. Since in one case or another there is a nonuniform stressed state close to the tip of a concentrator or a crack, then a certain form of the interconnection of the gradient approach with classical fracture mechanics is found in [13].

In the present work specific forms are considered for gradient criteria of strength which satisfy the condition of a connection with classical fracture mechanics and a combined version of them is also formulated. On the example of different stress concentration problems it is shown that use of gradient criteria of strength in the particular case of stress concentrators in the form of a crack leads to linear fracture mechanics relationships.

It is well known that for linear fracture mechanics equations there are certain application limits in the range of short crack lengths. Similar limitations are also found for gradient criteria in the case of small stress concentrators of the hole and pore type. The limitations obtained are then used by the Griffiths procedure in order to estimate critical defect dimensions in the form not only of microcracks, but also through holes and pores. These estimates are of undoubted interest since the strength of some brittle materials, including ceramic materials, is determined by presence of defects of the pore type.

1. Gradient Approach for Estimating Material Strength in the Zone of Stress Concentration. The maximum value of the first principal stress in a body $\max\sigma_1$ at the instant of the start of failure is called the local strength limit σ_* which is not a constant value and it depends on the degree of stressed state nonuniformity in the vicinity of a very critical point of a body. This nonuniformity may be specified by the relative gradient of the first principal stress

$$g_1 = |\mathrm{grad}\,\sigma_1|/\max\sigma_1, \tag{1.1}$$

which is calculated at a very critical point of a structural element from the elastic solution of the corresponding problem.

It is shown by experiment in [1, 5, 6] that the effect of a local increase in strength in the stress concentration zone may be described by a functional dependence $\sigma_* = \sigma_f f(g_1)$. Here σ_f is normal ultimate strength determined from smooth specimens whose cross-sectional area equals that of the specimen net cross section with a concentrator. The form of function $f(g_1)$ is found by proceeding from specific experimental data. It is noted in [6] that the relationship $\sigma_* = \sigma_f(1 + Bg_1^n)$ satisfactorily describes experimental results obtained for glass-reinforced plastics AG-4s and 33-18s.

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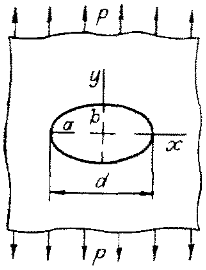

Fig. 1

In [12] attention is drawn to the fact that only with n = 1/2 for structural elements with concentrators in the form of cracks will finite values of limiting nominal stresses be obtained. In the opposite case a body with a crack is either impossible to break or it fails with zero loads. This means that the gradient strength criterion satisfying the requirement of infiniteness for the failure zone in for a concentrator of the crack type may be described as follows:

$$\sigma_* = \sigma_f(1 + \sqrt{L_1 g_1}) \tag{1.2}$$

(L_1 is a parameter having a dimension of length and depending on material properties, i.e., characteristic size).

In [13] using the well-known solution of the problem for extension of a plate with an elliptical hole (Fig. 1) it is shown that parameter L_1 should be connected with material crack resistance characteristics, i.e., critical stress intensity factor K_{1c}, by the relationship

$$L_1 = (2/\pi)\, K_{1c}^2/\sigma_f^2. \tag{1.3}$$

In this case for a concentrator in the form of a crack Griffiths criterion (1.2) gives an equation known in fracture mechanics for determining the limiting nominal stresses:

$$p_* = K_{1c}\sqrt{(2/\pi)/d} \tag{1.4}$$

(d is overall crack length).

However, the form of (1.2) considered previously for the gradient criterion satisfying the requirement of infiniteness of the failure load for a concentrator of the crack type is not the only one possible. For example, considering gradient models in the region of fatigue failure for structural elements [8, 9] the gradient strength criterion may also be written in the form

$$\sigma_* = \sigma_f\sqrt{1 + L_1 g_1}. \tag{1.5}$$

Since for concentrators of the crack type $g_1 \to \infty$, then criterion (1.5) also gives accurately the same results as (1.2). However, for concentrators which differ from cracks criteria (1.2) and (1.5) will give different results. Thus, with the same material characteristics σ_f and L_1 there are two different gradient strength criteria. There is no visible advantage of one criterion over the other. Experimental results for the failure of specimens with concentrators as a rule are found in the region between curves plotted by criteria (1.2) and (1.5). Therefore, a combined version of the gradient strength criterion is suggested:

$$\sigma_* = \sigma_f(1 - \beta + \sqrt{\beta^2 + L_1 g_1}). \tag{1.6}$$

Here β is a variable parameter ($\beta \geq 0$). If $\beta = 0$, then the combined criterion (1.6) is converted into (1.2), and if $\beta = 1$ then it is converted into (1.5).

For experimental results obtained for a specific material with a nonuniform stressed state it is possible to find a value of β with which the combined criterion (1.6) will describe these results better than (1.2) and (1.5). It is noted that according to [14] in order to describe the strength properties of a material around a hole it is sufficient to introduce two additional parameters one of which should have a dimension of length, and the second may be dimensionless. From criterion (1.6) the role of the first parameter is fulfilled by L_1, and the second by β. Parameter L_1 is determined by Eq. (1.3) from the

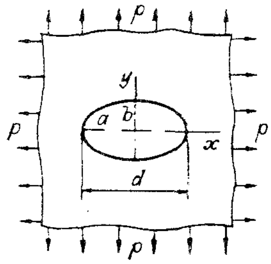

Fig. 2

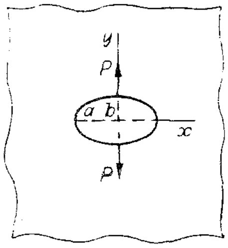

Fig. 3

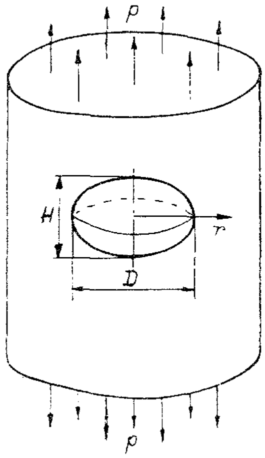

Fig. 4

condition for a link-up of the combined gradient strength criterion with linear fracture mechanics, and in essence β is introduced as an approximation parameter whose physical sense is not yet clear.

It is also noted that for concentrators in the form of cracks the combined criterion (1.6) will give the same equations as (1.2) and (1.5). Here the question arises: will they coincide with linear fracture mechanics equations not only in the case of a concentrator of the Griffiths crack type, but also in the general case?

2. Confirmation of the Connection of Gradient Strength Criteria with Linear Fracture Mechanics. In order to confirm the interconnection of gradient criteria with linear fracture mechanics and the validity in the general case of Eq. (1.3) connecting parameter L_1 with crack resistance K_{1c} it is possible to use known elastic solutions of the problem of stress concentration which permit at the limit conversion to stress concentrators in the form of cracks.

Examples of Plane Problems. First we consider the problem of biaxial tension of a plate with an elliptical hole (Fig. 2) whose solution is well known. In [15] there are equations which give the distribution of the first principal stress σ_1 over the critical cross section, i.e., over axis x:

$$\sigma_1 = p\frac{x(x^2 - a^2 + 2b^2)}{(x^2 - a^2 + b^2)^{3/2}}.$$

The maximum of σ_1 is reached with x = a. Consequently, stress concentration factor α, which is the ratio of maximum stress to niminal stress, is determined by the equation $\alpha = 2a/b$. It is noted that $\alpha \geq 2$ since always $a \geq b$. In view of symmetry of the problem $|\text{grad}\ \sigma_1| = |d\sigma_1/dx|$. Considering that $a = d/2$ and $\alpha \geq 2$ we obtain $|\text{grad}\ \sigma_1| = \alpha p(\alpha^2 - 2)/d$.

Since $\max\sigma_1 = \alpha p$, then from (1.1) we find $g_1 = (\alpha^2 - 2)/d$. By substituting this expression in the combined criterion we obtain

$$\sigma_* = \sigma_f(1 - \beta + \sqrt{\beta^2 + (\alpha^2 - 2)L_1/d}).$$

The nominal failure stress $p_* = \sigma_*/\alpha$, i.e.,

$$p_* = \sigma_f\left(\frac{1-\beta}{\alpha} + \frac{1}{\alpha}\sqrt{\beta^2 + (\alpha^2 - 2)L_1/d}\right).$$

At the limit with $\alpha \to \infty$ for a concentrator of the crack type and with finite values of parameter β we have

$$p_* = \sigma_f \sqrt{L_1/d}. \tag{2.1}$$

Substitution of (1.3) in (2.1) gives

$$p_* = K_{1c} \sqrt{(2/\pi)/d}. \tag{2.2}$$

An equation known in fracture mechanics was obtained in [15] for determining the nominal failure stress with biaxial tension of a plate with a rectilinear cut.

Thus, the correctness of Eq. (1.3) connecting L_1 with crack resistance K_{1c} was confirmed. In addition, with the problem in question of biaxial tension of a plate with an elliptical hole in essence was carried out by testing the results [13] since Eqs. (1.4) and (2.2) should coincide, and they did.

Now we consider the problem of the effect of concentrated forces on the contour of an elliptical hole in an infinite plate (Fig. 3). There is a solution for this problem in [16] where it is given in a special complex region and in a complex function of stresses. By using the Kolosov equation it is possible to find an expression for the stressed state component. Then it is necessary to change over to real coordinates of the problem and to describe the distribution of the first principal stress $\sigma_1 = \sigma_1(x)$ for the critical cross section. The maximum stress is achieved at the contour of an elliptical hole with x = a, it does not depend on the dimension of the major axis of the ellipse and it is determined by the expression

$$\max \sigma_1 = \frac{2P}{\pi b}. \tag{2.3}$$

In view of symmetry of the problem $|\mathrm{grad}\ \sigma_1| = |d\sigma_1/dx|$. After differentiating function $\sigma_1(x)$ with respect to coordinate x we determine $|\mathrm{grad}\ \sigma_1|$ with x = a. By substituting $|\mathrm{grad}\ \sigma_1|$ and $\max \sigma_1$ in expression (1.1) we obtain the equation sought for the relative gradient g_1 at the tip of a stress concentrator:

$$g_1 = 2a/b^2. \tag{2.4}$$

Substitution of (2.4) in (1.6) gives

$$\sigma_* = \sigma_f(1 - \beta + \sqrt{\beta^2 + 2L_1 a/b^2}). \tag{2.5}$$

At the instant of the start of failure $\max \sigma_1$ reaches a value of the local strength limit σ_*. From this condition by equating expressions (2.3) and (2.5) in order to determine concentrated forces at the instant of the start of failure we find an equation

$$P_* = \frac{\pi}{2} b\sigma_f(1 - \beta + \sqrt{\beta^2 + 2L_1 a/b^2}).$$

In the case of a concentrator of the crack type b → 0, i.e., with finite values of β we have

$$P_* = \sqrt{\frac{\pi^2}{2} \sigma_f^2 L_1 a}. \tag{2.6}$$

Substitution of (1.3) in (2.6) gives

$$P_* = K_{1c} \sqrt{\pi a}.$$

The equation obtained is known in fracture mechanics [15] for determining the value of concentrated forces which operate at the center in the edge of a crack at the instant of the start of its propagation. This means that once more the interconnection of gradient strength criteria with linear fracture mechanics was confirmed.

Example of a Spatial Problem. Now, in order to confirm the correctness of Eq. (1.3) we consider a spatial problem of the distribution of stresses around an axisymmetrical, flattened over the axis of symmetry, spheroidal cavity in an unbounded body with axial tension of this body along the axis of symmetry (Fig. 4). A solution of the problem is known [17] and it is given on elliptical coordinates. According to this solution the distribution of the first principal stress σ_1 over the critical cross section is found from the expression

$$\sigma_1 = p\left(1 + \frac{A+4B}{\mathrm{sh}^3(u)} + (12B + 2(1-\nu)C)\left(\mathrm{arctg}\left(\frac{1}{\mathrm{sh}(u)}\right) - \frac{1}{\mathrm{sh}(u)}\right)\right).$$

Here

$$A = VW^4[(6-8\nu)F - 6FW^2 + 4W^4 - 8(1-\nu)W^2]/N;$$
$$B = VW^4[2\nu F + 2W^4 - (1+2\nu)W^2]/N; \quad C = VW^4[6F - 12W^2]/N;$$
$$u = \ln\left(2W\frac{r}{D} + \sqrt{\left(2W\frac{r}{D}\right)^2 - 1}\right);$$
$$N = 6[8FW^2 - 2(1+\nu)F^2 - 6FW^4 + 4W^6 - 4W^4];$$
$$F = W^4V\,\mathrm{arctg}\left(\frac{1}{V}\right) - W^2V^2; \quad V = \sqrt{\frac{H^2}{D^2 - H^2}}; \quad W = \sqrt{\frac{D^2}{D^2 - H^2}};$$

D is cavity diameter in the critical section; H is cavity size over the axis of symmetry (height); r is current radius over the size of symmetry; and ν is Poisson's ratio.

Maximum stresses in the critical cross section are reached at the surface of a spheroidal cavity where $r = D/2$. It is possible to prove that at these points $\mathrm{sh}(u) = V$. This means that

$$\alpha = 1 + \frac{A+4B}{V^3} + (12B + 2(1-\nu)C)\left(\mathrm{arctg}\left(\frac{1}{V}\right) - \frac{1}{V}\right). \tag{2.7}$$

For determining (1.1) we find

$$g_1 = \frac{|24B + 4(1-\nu)C - 6(A+4B)W^2/V^2|}{A + 4B - (12B + 2(1-\nu)C)\left(V^2 - V^3\,\mathrm{arctg}\left(\frac{1}{V}\right)\right) + V^3}\frac{1}{D}. \tag{2.8}$$

We consider the limiting change-over from a spheroidal cavity to a flat circular crack with $H/D \to 0$, and in this case $V \to 0$, $W \to 1$. We write the combination of coefficients A, B, and C encountered in (2.7) and (2.8) only considering values of a higher order:

$$A + 4B = \frac{2}{\pi}V^2, \; 12B + 2(1-\nu)C = -\frac{2}{\pi}. \tag{2.9}$$

Substitution of (2.9) in (2.7) for α with $V \to 0$ gives the expression

$$\alpha = (4/\pi)/V. \tag{2.10}$$

Similarly by substituting (2.9) in (2.8) and considering in the denominator only terms of the order of smallness V^2 we obtain an expression for the relative gradient g_1 in the case of a concentrator of the flat circular crack type, i.e., with $H/D \to 0$, $V \to 0$, $W \to 1$:

$$g_1 = \frac{4}{V^2 D}. \tag{2.11}$$

Now we determine the nominal failure stress $p_* = \sigma_*/\alpha$. Taking account of (2.10) we write $p_* = (\pi/4)V\sigma_*$. Use of the combined criterion and expression (2.11) gives

$$p_* = \frac{\pi}{4}V\sigma_f\left(1 - \beta + \sqrt{\beta^2 + (4/V^2)L_1/D}\right).$$

Since $V \to 0$, then

$$p_* = \sigma_f\sqrt{(\pi^2/4)L_1/D}.$$

Considering (1.3) we have

$$p_* = K_{1c}\sqrt{(\pi/2)/D}. \tag{2.12}$$

The equation obtained was known in fracture mechanics for determining nominal failure stresses with the presence of a flat circular crack of diameter D.

Thus, use of gradient strength criteria in a particular case of stress concentrators in the form of cracks leads to relationships of linear fracture mechanics. However, for unsymmetrical problems the question of the conformity of results obtained by equations of classical fracture mechanics and by the gradient criteria requires further study.

3. The Case of Small Sized Stress Concentrators. It is well known that for linear fracture mechanics equations there are certain limits of applicability in the region of short

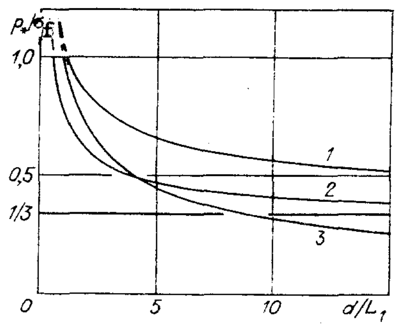

Fig. 5

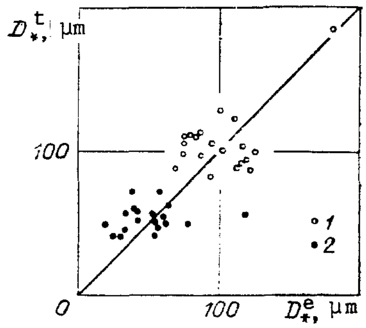

Fig. 6

length cracks. Similar limitations also correspond for gradient criteria with small sized stress concentrators in the form of holes and pores. In fact, in using gradient criteria in this case the nominal failure stress p_* will be greater than σ_f, which contradicts physical sense. For example, for the problem of uniaxial tension of a plate with an elliptical hole shown in Fig. 5 is the dependence of p_* on concentrator size over the critical cross section d for a Griffiths crack (curve 3) and for a circular hole (curves 1 and 2 plotted by means of criteria (1.2) and (1.5) respectively).

We consider more carefully the condition

$$p_* \leqslant \sigma_f \tag{3.1}$$

with the aim of finding the limits of consistency for the combined gradient criterion, for example, for concentrators in the form of through elliptical holes (see Fig. 1). Here the relative gradient is found from an equation [13]

$$g_1 = (\alpha - 1)^2(1 + 1/2\alpha)/d,$$

where $\alpha = 1 + 2a/b$. After substituting this equation in criterion (1.6) for determining $p_* = \sigma_*/\alpha$ we have

$$p_* = \sigma_f\left(\frac{1-\beta}{\alpha} + \frac{1}{\alpha}\sqrt{\beta^2 + (\alpha-1)^2(1+1/2\alpha)L_1/d}\right).$$

By using condition (3.1) and an expression for p_* we obtain the limitation sought $d \geq d_*$, where

$$d_* = \frac{\alpha - 1}{\alpha + 2\beta - 1}\left(1 + \frac{1}{2\alpha}\right)L_1. \tag{3.2}$$

Thus, gradient criterion (1.6) with a specified value of β may only be used when the size of the hole over the critical cross section is not less than a certain value of d_* for a given hole shape. Otherwise we obtain high values of calculated limiting loads which contradict common sense.

The limitation found is similar to those which arise in linear fracture mechanics with small crack sizes and similarly it may be used in order to evaluate the critical size of defects in a material in the form not only of microcracks, but also through holes. In fact, the presence in a material of defects of certain (critical) sizes explains in Griffiths theory the real strength σ_f of brittle materials. It is noted that with $\alpha \to \infty$, according to (3.2), we have $d_* = L_1$, i.e., parameter L_1 appeared to equal the critical size of a defect of the Griffiths crack type.

As is well known, the strength of some brittle materials, including ceramic materials, is determined by the presence of defects in the form of pores. In [18-21] it is shown by experiment that the larger the size of defects of this type, the less is material strength. It is necessary to estimate the limiting permissible defect size in the form of pores. Let there be spherical pores. In this case it is possible to use the Leon solution for stress distribution around a spherical cavity in an unbounded body with uniaxial tension for this body. According to the Leon solution [16] there is the following distribution of first principal stress through the critical cross section:

$$\sigma_1 = p\left(1 + \frac{4-5\nu}{14-10\nu}\frac{a^3}{r^3} + \frac{9}{14-10\nu}\frac{a^5}{r^5}\right).$$

Here p is nominal stress; ν is Poisson's ratio. The maximum stresses are achieved at the surface of a spherical cavity where r = a. This means that

$$\alpha = \frac{27-15\nu}{14-10\nu}. \tag{3.3}$$

In view of symmetry of the problem $|\mathrm{grad}\,\sigma_1| = \left|\frac{d\sigma_1}{dr}\right| = \frac{57-15\nu}{14-10\nu}\frac{p}{a}$. In accordance with (1.1) we find that

$$g_1 = \frac{57-15\nu}{27-15\nu}\frac{1}{a}. \tag{3.4}$$

Use of combined gradient criterion (1.6) considering (3.4) and (3.3) in order to determine the nominal failure stress gives an equation

$$p_* = \frac{14-10\nu}{27-15\nu}\sigma_f\left(1 - \beta + \sqrt{\beta^2 + \frac{114-30\nu}{27-15\nu}L_1/D}\right),$$

where D = 2a is spherical cavity diameter. By using this equation and the condition $p_* = \sigma_f$ we find the critical diameter of a defect in the form of a spherical pore:

$$D_* = \frac{114-30\nu}{27-15\nu}\frac{L_1}{(\alpha-1)(\alpha+2\beta-1)}. \tag{3.5}$$

Here concentration factor α is determined by Eq. (3.3).

For example, with $\nu = 0.22$ for ceramic WC–10% Co [22] assuming that $\beta = 1/2$ from Eq. (3.5) we have $D_* = 2.237\ L_1$. Use of this estimate applied to experimental data [20] obtained in WC–10% Co material leads to good agreement of theoretical D_*^t and experimental D_*^e values (Fig. 6). Brittle failure of specimens is initiated by defects in the form of artificial or natural pores (points 1 and 2) whose diameter after breakage is found by means of a scanning electron microscope.

In [20, 21] it is assumed that action of a spherical pore of certain diameter on the strength of an equivalently affected flat circular crack of exactly the same diameter, i.e., in accordance with (2.12) and (1.3)

$$D_* = \frac{\pi^2}{4}L_1 \approx 2{,}47\,L_1.$$

In addition, in order to estimate the critical sizes of defects in the form of pores there is use in [18, 19] of a the model of a pore surrounded by an annular crack. According to estimates in [19] for satisfactory description of experimental data for brittle failure of silicon nitride the length of annular cracks should be an order of magnitude greater than the grain size, i.e., these cracks should have a length sufficient for their detection. However, experimental studies carried out in [20] by means of a scanning electron microscope did not confirm this hypothesis about the presence of annular cracks around pores. In this connection it should be noted that use of the combined gradient strength criterion (1.6) makes it possible to obtain suitable estimates of critical defect sizes in the form of pores without drawing on additional hypotheses about the presence of annular cracks around pores.

Attention is drawn once again to the fact that for concentrators of the crack type this criterion gives linear fracture mechanics equations confirmed in practice. Thus, analysis of the suggested combined gradient strength criterion (1.6) demonstrated its broad universal possibilities and promising nature for use in strength analysis.

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